Logistics Network Design and Facility Location: The value of a multi-period stochastic solution

Francisco Saldanha-da-Gama

Departmento de Estatística e Investigação Operacional,
Centro de Matemática Aplicações Fundamentais e Investigação Operacional,
Faculdade de Ciências, Universidade de Lisboa, Portugal

ORBEL/GOR Joint Annual International Conference

Brussels, Belgium, September 12–14, 2008

Outline

Outline

A facility location problem consists of determining the “best” location for one or several facilities or equipments in order to serve a set of demand points.

Application Areas:

- Telecommunications,
- Urban planning,
- Layout problems,
- Quantitative Marketing,
- Logistics,
- *et cetera*.

A facility location problem consists of determining the “best” location for one or several facilities or equipments in order to serve a set of demand points.

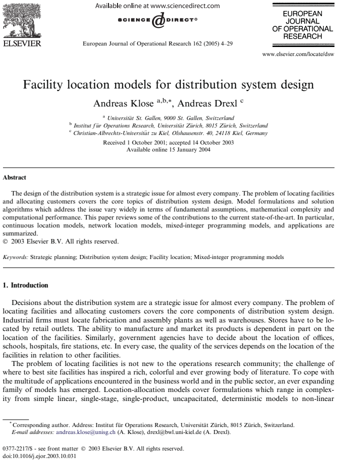
Application Areas:

- Telecommunications,
- Urban planning,
- Layout problems,
- Quantitative Marketing,
- Logistics,
- *et cetera*.

Outline

Historically, researchers have focused relatively early on the role of facility location when designing logistics networks.

- capacity decisions;
- multiple layers of facilities;
- multiple products;
- multiple objectives;
- ....



ELSEVIER
Available online at www.sciencedirect.com
SCIENCE DIRECT®
European Journal of Operational Research 162 (2005) 4–29
EUROPEAN JOURNAL OF OPERATIONAL RESEARCH
www.elsevier.com/locate/dsw

Facility location models for distribution system design

Andreas Klose [a,b,*], Andreas Drexl [c]

[a] *Universität St. Gallen, 9000 St. Gallen, Switzerland*
[b] *Institut für Operations Research, Universität Zürich, 8015 Zürich, Switzerland*
[c] *Christian-Albrechts-Universität zu Kiel, Olshausenstr. 40, 24118 Kiel, Germany*

Received 1 October 2001; accepted 14 October 2003
Available online 15 January 2004

Abstract

The design of the distribution system is a strategic issue for almost every company. The problem of locating facilities and allocating customers covers the core topics of distribution system design. Model formulations and solution algorithms which address the issue vary widely in terms of fundamental assumptions, mathematical complexity and computational performance. This paper reviews some of the contributions to the current state-of-the-art. In particular, continuous location models, network location models, mixed-integer programming models, and applications are summarized.

Keywords: Strategic planning; Distribution system design; Facility location; Mixed-integer programming models

1. Introduction

Decisions about the distribution system are a strategic issue for almost every company. The problem of locating facilities and allocating customers covers the core components of distribution system design. Industrial firms must locate fabrication and assembly plants as well as warehouses. Stores have to be located by retail outlets. The ability to manufacture and market its products is dependent in part on the location of the facilities. Similarly, government agencies have to decide about the location of offices, schools, hospitals, fire stations, etc. In every case, the quality of the services depends on the location of the facilities in relation to other facilities.

The problem of locating facilities is not new to the operations research community; the challenge of where to best site facilities has inspired a rich, colorful and ever growing body of literature. To cope with the multitude of applications encountered in the business world and in the public sector, an ever expanding family of models has emerged. Location-allocation models cover formulations which range in complexity from simple linear, single-stage, single-product, uncapacitated, deterministic models to non-linear

* Corresponding author. Address: Institut für Operations Research, Universität Zürich, 8015 Zürich, Switzerland.
E-mail addresses: andreas.klose@unisg.ch (A. Klose), drexl@bwl.uni-kiel.de (A. Drexl).

0377-2217/$ - see front matter © 2003 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2003.10.031

European Journal of Operational Research 196 (2009) 401–412

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Invited Review

Facility location and supply chain management – A review

M.T. Melo [a,b], S. Nickel [b,c], F. Saldanha-da-Gama [d,*]

[a] Department of Business Administration, University of Applied Sciences, D 66123 Saarbrücken, Germany
[b] Department of Optimization, Fraunhofer Institute for Industrial Mathematics (ITWM), D 67663 Kaiserslautern, Germany
[c] Chair of Operations Research and Logistics, Saarland University, D 66041 Saarbrücken, Germany
[d] Department of Statistics and Operations Research and Operations Research Center, University of Lisbon, P 1749-016 Lisbon, Portugal

ARTICLE INFO

Article history:
Received 4 December 2007
Accepted 4 May 2008
Available online 16 May 2008

Keywords:
Facility location
Supply chain management
Network design

ABSTRACT

Facility location decisions play a critical role in the strategic design of supply chain networks. In this paper, a literature review of facility location models in the context of supply chain management is given. We identify basic features that such models must capture to support decision-making involved in strategic supply chain planning. In particular, the integration of location decisions with other decisions relevant to the design of a supply chain network is discussed. Furthermore, aspects related to the structure of the supply chain network, including those specific to reverse logistics, are also addressed. Significant contributions to the current state-of-the-art are surveyed taking into account numerous factors. Supply chain performance measures and optimization techniques are also reviewed. Applications of facility location models to supply chain network design ranging across various industries are presented. Finally, a list of issues requiring further research are highlighted.

1. Introduction

Facility location is and has been a well established research area within Operations Research (OR). Numerous papers and books are witnesses of this fact (see, e.g. [29] and references therein). The American Mathematical Society (AMS) even created specific codes for location problems (90B80 for discrete location and assignment, and 90B85 for continuous location). Nevertheless, the question of the applicability of location models has always been under discussion. In contrast, the practical usefulness of logistics was never an issue. One of the areas in logistics which has attracted much attention is Supply Chain Management (SCM) (see, e.g. [114] and references therein). In fact, the development of SCM started independently of OR and only step by step did OR enter into SCM (see, e.g. [18]). As a consequence, facility location models have been gradually proposed within the supply chain context (including reverse logistics), thus opening an extremely interesting and fruitful application domain. There are naturally several questions which immediately arise during such a development, namely: (i) What properties does a facility location model have to fulfill to be acceptable within the supply chain context? (ii) Are there existing facility location models which already fit into the supply chain context? (iii) Does SCM need facility location models at all?

As the number of papers has increased tremendously in the last few years and even the Association of European Operational Research Societies (EURO) has recently devoted a Winter institute to this topic [35], we felt that the time was ripe to have a review paper looking exactly at the role of facility location models within SCM. Before starting the review we briefly define our two main objects of investigation, namely facility location and SCM.

A general facility location problem involves a set of spatially distributed customers and a set of facilities to serve customer demands (see, e.g. [29,90]). Moreover, distances, times or costs between customers and facilities are measured by a given metric (see [96]). Possible questions to be answered are: (i) Which facilities should be used (opened)? (ii) Which customers should be serviced from which facility (or facilities) so as to minimize the total costs? In addition to this generic setting, a number of constraints arise from the specific application domain. For recent reviews on facility location we refer to Klose and Drexl [58] and ReVelle et al. [97].

SCM is the process of planning, implementing and controlling the operations of the supply chain in an efficient way. SCM spans all movements and storage of raw materials, work-in-process inventory, and finished goods from the point-of-origin to the point-of-consumption (see [114] and the Council of Supply Chain Management Professionals [21]). Part of the planning processes in SCM aims at finding the best possible supply chain configuration. In addition to the generic facility location setup, also other areas such as procurement, production, inventory, distribution, and routing have to be considered (see [20]). Historically, researchers have focused relatively early on the design of distribution systems (see [58] and references therein) but without considering the supply chain as a whole.

* Corresponding author. Tel.: +351 217500010; fax: +351 217500081.
E-mail address: fsgama@fc.ul.pt (F. Saldanha-da-Gama).

0377-2217/$ - see front matter © 2008 Elsevier B.V. All rights reserved.
doi:10.1016/j.ejor.2008.05.007

Two aspects of major relevance:

TIME UNCERTAINTY

Time: “amount” of future to consider.

- inventory decisions;
- capacity adjustments;
- opening/closing facilities;
- ...

Uncertainty: degree and type of knowledge available about future developments.

- demand levels;
- capacities;
- ...

European Journal of Operational Research 264 (2018) 813–829

Contents lists available at ScienceDirect

European Journal of Operational Research

journal homepage: www.elsevier.com/locate/ejor

Time traps in supply chains: Is optimal still good enough?

Fabian Dunke[a], Iris Heckmann[b], Stefan Nickel[a,b,*], Francisco Saldanha-da-Gama[c]

[a] Institute of Operations Research, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany
[b] Department of Logistics and Supply Chain Optimization, Research Center for Information Technology (FZI), Karlsruhe, Germany
[c] DEIO and CMAF-CIO, Faculdade de Ciências, Universidade de Lisboa, Lisboa, Portugal

ARTICLE INFO

Article history:
Received 4 January 2016
Accepted 9 July 2016
Available online 16 July 2016

Keywords:
Optimality concepts
Online optimization
Look-ahead
Supply chain risk
Multi-period planning

ABSTRACT

Operations Researchers support Supply Chain Management and Supply Chain Planning by developing adequate mathematical optimization models and providing suitable solution procedures. In this paper we discuss what *adequate* could mean. Therefore, we may ask several questions concerning "optimality" in Supply Chain Planning under causal and temporal uncertainty: What is an optimal solution? When is it optimal? For how long is it optimal? How should the design of a supply chain be changed when conditions and requirements ask for new structures? In particular, we discuss new approaches to Supply Chain Planning in order to give an optimal transformation from an initial solution over multiple periods to a desired one rather than just specifying an optimal snapshot solution. Time and uncertainty are the factors triggering the whole discussion. In particular, several flaws often found when dealing with these factors result in so-called "time traps". We look at the impact of recent technological developments like the Internet of Things or Industry 4.0 on operational supply chain planning and control, and we show how online optimization can help to cope with real-time challenges. Moreover, we re-coin the concept of risk in the realm of Supply Chain Planning. Here the question is how to measure supply chain specific risks and how to incorporate them "adequately" into mathematical models.

1. Introduction

Supply Chain Planning—as an important subtask of Supply Chain Management—is the process of allocating resources over a network of interrelated locations with the goal to satisfy customer requirements. It spans all movements and storage of raw materials, work-in-process inventory, and finished goods from the point-of-origin to the point-of-consumption. Operations Researchers support Supply Chain Planning by developing mathematical optimization models and providing suitable solution procedures.

The concept of *optimality* describes the property of a solution which imposes the best feasible decision obtainable under specific conditions. These conditions need to be identified, gathered, and appropriately expressed by formulating mathematical models, which abstract from restrictions of the real world. If models do not capture the most relevant features and do not yield to applicable tasks or useful managerial insights, their solutions will never be regarded as good enough for practical implementation—although they are optimal from a strictly mathematical point of view.

Especially global supply chains have to face a rich variety of potential requirements. Not all of them can be considered within constraints, but some of them must be respected. Since Supply Chain Planning strongly depends on the ability to grasp future developments in order to balance supply and demand, the main challenge during the identification of important requirements is imposed by the weighting and the incorporation of characteristics that describe the future. Major components of the future are time and uncertainty. While the former refers to the "amount" of future to consider, the latter describes the degree and type of knowledge available about future developments. Ignoring an appropriate way of dealing with these two aspects—isolated or together–leads to what we call "time traps", which is a term indicating that the relevance of time is perceived but not adequately treated.

In this paper we claim that the existing optimization models for supporting Supply Chain Planning lack to address the future appropriately and thus, do not assure that optimal solutions represent applicable plans and provide intelligible benefits. We address three major topics that overlap with respect to their treatment of the future, namely: online optimization models, multi-period planning models, and risk-aware models.

* Corresponding author at: Institute of Operations Research, Karlsruhe Institute of Technology (KIT), Karlsruhe, Germany. Fax: +49 72160843951.
E-mail addresses: fabian.dunke@kit.edu (F. Dunke), heckmann@fzi.de (I. Heckmann), stefan.nickel@kit.edu (S. Nickel), fsgama@ciencias.ulisboa.pt (F. Saldanha-da-Gama).

http://dx.doi.org/10.1016/j.ejor.2016.07.016

- Uncertainty leads to "risk".

  ↓

  But what is risk?...

  How can we quantify it?...

  How to build a "risk-aware" facility location model?...

- Major components of future:

  TIME UNCERTAINTY

- Handling short term future uncertainty can be accomplished by online optimization.

  [Dunke and Nickel, Omega, 2016]

  [Dunke and Nickel, CEJOR, 2017]

  [Dunke, et al., EJOR, 2018]

  [Dunke and Nickel, J Simulation, 2018]

- This presentation:

  $\downarrow$

  Strategic logistics network design: mid- to long-term future.

Outline

Notation

I set of potential locations for the facilities.

J set of customers/demand points.

f_i operation cost for facility $i \in I$.

c_{ij} unit transportation cost between facility $i \in I$ and customer $j \in J$.

d_j demand of customer $j \in J$.

q_i capacity of facility $i \in I$.

Decision variables

$$y_i = \begin{cases} 1, & \text{if facility } i \in I \text{ is open;} \\ 0, & \text{otherwise.} \end{cases}$$

x_{ij} = fraction of the demand of customer $j \in J$ supplied from facility $i \in I$.

The capacitated facility location problem

$$
\begin{aligned}
\min \quad & \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} c_{ij} d_j x_{ij} & \\
s.t. \quad & \sum_{i \in I} x_{ij} = 1 & j \in J \\
& \sum_{j \in J} d_j x_{ij} \le q_i y_i & i \in I \\
& y_i \in \{0, 1\} & i \in I \\
& x_{ij} \ge 0 & i \in I,\ j \in J
\end{aligned}
$$

Outline

Multi-period planning

When should we consider time explicitly in a facility location problem?

- It is possible and desirable to adapt/change location decisions throughout time;
- We observe changes over time in parameters such as costs and demand levels and we are able to model those changes;
- Other decisions need to be made that require time to be explicitly considered (e.g. investment, inventory);
- Capacity adjustments can be made;
- ...

[Nickel and Saldanha-da-Gama, LS, 2015]

Multi-period planning

In (dynamic) facility location problems time is often discretized. Why?

- The models are easier to handle...

  Typically, decision variables can be associated with the different periods of the planning horizon.

  $\downarrow$

  A mixed-integer mathematical programming model can often be derived.

- The organization of the data makes multi-period models more natural.

  For instance, we often find or look for daily, weekly or monthly demand levels.

- To a large extent forecasting systems typically work with time periods no matter their length.

Multi-period planning

Length of a time period?

- Is primarily determined by the decisions to be planned.
- Depending on the information we have, the length of a time period can be easily adjusted.

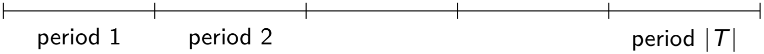
  $\downarrow$

  If we have more detailed information we can consider a daily planning; otherwise we can go into a monthly or yearly planning, for instance.

Multi-period planning

T set of time periods.

Planning horizon:



There as specific moments for implementing changes/decisions.

Outline

Modeling aspects

Adequate model?

[Current et al., EJOR, 1997]:

- Implicit dynamic (multi-period) problem.

  All facilities opened at the beginning of the planning horizon.

  The selected locations account for the effect of the time dependent parameters.

- Explicit dynamic (multi-period) problem.

  Facilities are opened and/or closed throughout the planning horizon.

  A plan is devised for opening/closing facilities at specific times and locations in response to changes in parameters over time.

Implicit multi-period location

Parameters:

- d_{jt} demand of customer $j \in J$ in period $t \in T$.
- c_{ijt} unit transportation cost between facility $i \in I$ and customer $j \in J$ in period $t \in T$.
- f_i fixed cost associated with facility $i \in I$.

Decision variables:

x_{ijt} = fraction of the demand of customer $j \in J$ in period $t \in T$ supplied from facility $i \in I$.

$$y_i = \begin{cases} 1, & \text{if facility } i \in I \text{ is installed;} \\ 0, & \text{otherwise.} \end{cases}$$

Implicit multi-period location

$$\min \quad \sum_{i \in I} f_i y_i + \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} c_{ijt} d_{jt} x_{ijt}$$

$$s.t. \quad \sum_{i \in I} x_{ijt} = 1 \qquad j \in J,\ t \in T$$

$$\sum_{j \in J} d_{jt} x_{ijt} \leq q_i y_i \qquad i \in I,\ t \in T$$

$$y_i \in \{0,1\} \qquad i \in I$$

$$x_{ijt} \geq 0 \qquad i \in I,\ j \in J,\ t \in T$$

Explicit multi-period location

Parameters:

- d_{jt} demand of customer $j \in J$ in period $t \in T$.
- c_{ijt} unit transportation cost between facility $i \in I$ and customer $j \in J$ in period $t \in T$.
- f_{it} cost for operating facility $i \in I$ in period $t \in T$.

Decision variables:

x_{ijt} = fraction of the demand of customer $j \in J$ supplied from facility $i \in I$ in period $t \in T$.

$$y_{it} = \begin{cases} 1, & \text{if facility } i \in I \text{ is open in period } t \in T; \\ 0, & \text{otherwise.} \end{cases}$$

Explicit multi-period location

$$
\begin{aligned}
\min \quad & \sum_{i\in I}\sum_{t\in T} f_{it} y_{it} + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} c_{ijt} d_{jt} x_{ijt} & \\
s.t. \quad & \sum_{i\in I} x_{ijt} = 1 & j \in J,\ t \in T \\
& \sum_{j\in J} d_{jt} x_{ijt} \le q_i y_{it} & i \in I,\ t \in T \\
& y_{it} \in \{0,1\} & i \in I,\ t \in T \\
& x_{ijt} \ge 0 & i \in I,\ j \in J,\ t \in T
\end{aligned}
$$

Explicit multi-period location

$$\min \quad \sum_{i \in I} \sum_{t \in T} f_{it} y_{it} + \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} c_{ijt} d_{jt} x_{ijt}$$

$$s.t. \quad \sum_{i \in I} x_{ijt} = 1 \qquad j \in J,\ t \in T$$

$$\sum_{j \in J} d_{jt} x_{ijt} \leq q_i y_{it} \qquad i \in I,\ t \in T$$

$$y_{it} \in \{0, 1\} \qquad i \in I,\ t \in T$$

$$x_{ijt} \geq 0 \qquad i \in I,\ j \in J,\ t \in T$$

Explicit multi-period location — A phase-in problem

$$\min \quad \sum_{i\in I}\sum_{t\in T} f_{it}y_{it} + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} c_{ijt}d_{jt}x_{ijt}$$

$$s.t. \quad \sum_{i\in I} x_{ijt} = 1 \qquad j\in J,\ t\in T$$

$$\sum_{j\in J} d_{jt}x_{ijt} \le q_i y_{it} \qquad i\in I,\ t\in T$$

$$y_{it} \le y_{i,t+1} \qquad i\in I,\ t=1,\ldots,|T|-1$$

$$y_{it}\in\{0,1\} \qquad i\in I,\ t\in T$$

$$x_{ijt}\ge 0 \qquad i\in I,\ j\in J,\ t\in T$$

Explicit multi-period location — Phase-in/phase-out

$$I = I^c \cup I^o.$$

I^c = set of locations where (existing) facilities can be removed.

I^o = set of locations where new facilities can be installed.

Explicit multi-period location — Phase-in/Phase-out

$$\min \quad \sum_{i\in I}\sum_{t\in T} f_{it}y_{it} + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} c_{ijt}d_{jt}x_{ijt}$$

$$
\begin{aligned}
s.t.\quad & \sum_{i\in I} x_{ijt} = 1 && j\in J,\ t\in T\\
& \sum_{j\in J} d_{jt}x_{ijt} \le q_i y_{it} && i\in I,\ t\in T\\
& y_{it} \le y_{i,t+1} && i\in I^o,\ t=1,\dots,|T|-1\\
& y_{it} \ge y_{i,t+1} && i\in I^c,\ t=1,\dots,|T|-1\\
& y_{it}\in\{0,1\} && i\in I,\ t\in T\\
& x_{ijt}\ge 0 && i\in I,\ j\in J,\ t\in T
\end{aligned}
$$

Explicit multi-period location — Reformulation

$$y_{it} = \begin{cases} 1, & \text{if facility } i \in I \text{ is open in period } t \in T; \\ 0, & \text{otherwise.} \end{cases}$$

Alternative [Van Roy and Erlenkotter, Mgmt Sci, 1982]:

$i \in I^o$, $t \in T$

$$z_{it} = \begin{cases} 1 & \text{if facility } i \text{ is installed at the beginning of period } t \\ 0 & \text{otherwise.} \end{cases}$$

$i \in I^c$, $t = 1, \ldots, |T| - 1$

$$z_{it} = \begin{cases} 1 & \text{if facility } i \text{ is removed at the end of period } t \\ 0 & \text{otherwise.} \end{cases}$$

STEP versus IMPULSE variables [Albareda-Sambola et al., TOP, 2010]

Outline

An explicit multi-period phase-in location problem with service level

In many logistics applications, a service level below 100% is acceptable or even desirable.

But...it is usually not for "free"...

v_{jt} proportion of the demand of customer $j \in J$ in period $t \in T$ that is unfulfilled.

r_{jt} unit cost for unfulfilled demand of customer $j \in J$ in period $t \in T$.

An explicit multi-period phase-in location problem with service level

$$\min \quad \sum_{i \in I} f_{it} y_{it} + \sum_{t \in T} \sum_{j \in J} \sum_{i \in I} d_{jt} c_{ijt} x_{ijt} + \sum_{t \in T} \sum_{j \in J} d_{jt} r_{jt} v_{jt}$$

$$\begin{aligned}
s.t. \quad & \sum_{i \in I} x_{ijt} + v_{jt} = 1 && j \in J,\ t \in T \\
& \sum_{j \in J} d_{jt} x_{ijt} \leq q_i y_{it} && i \in I,\ t \in T \\
& y_{it} \leq y_{i,t+1} && i \in I,\ t = 1, \ldots, |T| - 1 \\
& y_{it} \in \{0, 1\} && i \in I,\ t \in T \\
& x_{ijt} \geq 0 && i \in I,\ j \in J,\ t \in T \\
& v_{jt} \geq 0 && j \in J,\ t \in T
\end{aligned}$$

An explicit multi-period phase-in location problem with service level

[Castro et al., Math Prog, 2017]

- Consider values $p_1 \leq p_2 \leq \ldots \leq p_{|T|} \leq |I|$ to capture a "maximum speed" for making adjustments in the set of operating facilities:

$$\sum_{i \in I} y_{it} \leq p_t, \quad t \in T$$

- Discuss the meaning of service level in a multi-period context:

Global service level: $\text{GSL} = \frac{\sum_{t \in T} \sum_{i \in I} \sum_{j \in J} d_{jt} x_{ijt}}{\sum_{t \in T} \sum_{j \in J} d_{jt}}.$

Average service level: $\text{ASL} = \frac{1}{|T|} \sum_{t \in T} SL(t), \quad SL(t) = \frac{\sum_{j \in J} \sum_{i \in I} d_{jt} x_{ijt}}{\sum_{j \in J} d_{jt}}$

An explicit multi-period phase-in location problem with service level

[Castro et al., Math Prog, 2017]

- Develop a Benders decomposition.

  A specialized interior-point method for linear programming problems with a block-angular structure is customized for solving the benders linear subproblems.

  Instances with up to 3 periods, 200 locations and 1.000.000 customers are solved to optimality.

An explicit multi-period phase-in location problem with service level

$$\min \quad \sum_{i\in I} f_{it} y_{it} + \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} d_{jt} c_{ijt} x_{ijt} + \sum_{t\in T}\sum_{j\in J} d_{jt} r_{jt} v_{jt}$$

$$\begin{aligned}
s.t. \quad & \sum_{i\in I} x_{ijt} + v_{jt} = 1 && j\in J,\ t\in T \\
& \sum_{j\in J} d_{jt} x_{ijt} \le q_i y_{it} && i\in I,\ t\in T \\
& y_{it} \le y_{i,t+1} && i\in I,\ t = 1,\ldots,|T|-1 \\
& y_{it} \in \{0,1\} && i\in I,\ t\in T \\
& x_{ijt} \ge 0 && i\in I,\ j\in J,\ t\in T \\
& v_{jt} \ge 0 && j\in J,\ t\in T
\end{aligned}$$

An explicit multi-period phase-in location problem with service level

$$
\begin{aligned}
\min \quad & \sum_{i\in I} f_{it} y_{it} + \sum_{t\in T}\sum_{j\in J}\sum_{i\in I} \left[c_{ijt} - r_{jt}\right] d_{jt} x_{ijt} \\
s.t. \quad & \sum_{i\in I} x_{ijt} \le 1 && j \in J,\ t\in T \\
& \sum_{j\in J} d_{jt} x_{ijt} \le q_i y_{it} && i\in I,\ t\in T \\
& y_{it} \le y_{i,t+1} && i\in I,\ t = 1,\dots,|T|-1 \\
& y_{it} \in \{0,1\} && i \in I,\ t\in T \\
& x_{ijt} \ge 0 && i\in I,\ j\in J,\ t\in T
\end{aligned}
$$

r_{jt} can be looked as as the unit revenue for selling to customer $j \in J$ in period $t \in T$.

Outline

Relevance of a multi-period modeling framework

How relevant is it to consider a multi-period modeling framework instead of a (more simplified) static one?

Suppose that we have the possibility of making a static location decision even with costs, demands and possibly other parameters varying over time.

Is it still worth considering a multi-period modeling framework?

An answer to this question can be given by the value of the multi-period solution.

[Alumur et al., EJOR, 2012]

[Nickel and Saldanha-da-Gama, LS, 2015].

Relevance of a multi-period modeling framework

Definition

The value of the multi-period solution is difference between the optimal value of the multi-period problem and the value of a (static) solution found by solving a static counterpart.

Definition

A static counterpart is a problem that takes into account the information available for the planning horizon and looks for a static (time-invariant) solution in terms of the location of the facilities.

Relevance of a multi-period modeling framework

Methodology:

- Find a static counterpart and solve it optimally;
- Check whether the resulting location decisions are feasible to the multi-period problem.

$\downarrow$

- $\surd$ Set such solution for all periods of the planning horizon.
- $\surd$ The difference between its value and the optimal value of the multi-period problem gives the value of the multi-period solution.

Relevance of a multi-period modeling framework

A static counterpart can be obtained by imposing that the status of a location does not change during the planning horizon.

→ Implicit multi-period location...

$$
\begin{aligned}
\min \quad & \sum_{i\in I} f_{it} y_{it} + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} c_{ijt} d_{jt} x_{ijt} & \\
s.t. \quad & \sum_{i\in I} x_{ijt} = 1 & j\in J,\ t\in T\\
& \sum_{j\in J} d_{jt} x_{ijt} \le q_i y_{it} & i\in I,\ t\in T\\
& y_{it} \le y_{i,t+1}, & i\in I^o,\ t=1,\dots,|T|-1\\
& y_{it} \ge y_{i,t+1}, & i\in I^c,\ t=1,\dots,|T|-1\\
& y_{it}\in\{0,1\} & i\in I,\ t\in T\\
& x_{ijt}\ge 0 & i\in I,\ j\in J,\ t\in T
\end{aligned}
$$

$$
\begin{aligned}
\min \quad & \sum_{i\in I}\sum_{t\in T} f_{it} y_i + \sum_{i\in I}\sum_{j\in J}\sum_{t\in T} c_{ijt} d_{jt} x_{ijt} & \\
s.t. \quad & \sum_{i\in I} x_{ijt} = 1 & j\in J,\ t\in T\\
& \sum_{j\in J} d_{jt} x_{ijt} \le q_i y_i & i\in I,\ t\in T\\
& y_i\in\{0,1\} & i\in I\\
& x_{ijt}\ge 0 & i\in I,\ j\in J,\ t\in T
\end{aligned}
$$

Relevance of a multi-period modeling framework

A static counterpart can be obtained by imposing that the status of a location does not change during the planning horizon.

$\rightarrow$ Implicit multi-period location...

$$
\begin{aligned}
\min \quad & \sum_{i \in I} f_{it} y_{it} + \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} c_{ijt} d_{jt} x_{ijt} \\
s.t. \quad & \sum_{i \in I} x_{ijt} = 1 && j \in J,\ t \in T \\
& \sum_{j \in J} d_{jt} x_{ijt} \leq q_i y_{it} && i \in I,\ t \in T \\
& y_{it} \leq y_{i,t+1}, && i \in I^o,\ t = 1, \ldots, |T| - 1 \\
& y_{it} \geq y_{i,t+1}, && i \in I^c,\ t = 1, \ldots, |T| - 1 \\
& y_{it} \in \{0, 1\} && i \in I,\ t \in T \\
& x_{ijt} \geq 0 && i \in I,\ j \in J,\ t \in T
\end{aligned}
$$

$$
\begin{aligned}
\min \quad & \sum_{i \in I} F_i y_i + \sum_{i \in I} \sum_{j \in J} \sum_{t \in T} c_{ijt} d_{jt} x_{ijt} \\
s.t. \quad & \sum_{i \in I} x_{ijt} = 1 && j \in J,\ t \in T \\
& \sum_{j \in J} d_{jt} x_{ijt} \leq q_i y_i && i \in I,\ t \in T \\
& y_i \in \{0, 1\} && i \in I \\
& x_{ijt} \geq 0 && i \in I,\ j \in J,\ t \in T
\end{aligned}
$$

Outline

Dealing with uncertainty

- Common sources of uncertainty in facility location problems emerging in the context of logistics network design:

demands capacities costs/revenues

- For the moment we consider deterministic and time-invariant costs/revenues.
- We assume that uncertainty is fully captured by a finite set of scenarios, S.

We first focus on stochastic demand and then we include uncertainty in capacities.

Notation

Sets

- I set of potential locations for the facilities.
- J set of customers.
- T set of periods in the planning horizon.
- S set of scenarios describing the uncertainty.

Costs/revenues

- f_i fixed cost for facility $i \in I$.
- c_{ij} unit transportation costs between location $i \in I$ and customer $j \in J$.
- r_j unit revenue associated with customer $j \in J$.

Other parameters

- q_i capacity of a facility operating at $i \in I$.
- d_{jts} demand of customer $j \in J$ in period $t \in T$ under scenario $s \in S$.
- π_s probability of scenario $s \in S$.

A first model capturing stochasticity

x_{ijts} = fraction of the demand of customer $j \in J$ in period $t \in T$ under scenario $s \in S$ supplied from facility $i \in I$.

$$
\begin{aligned}
\min \quad & \sum_{i \in I} f_i y_i + \sum_{s \in S} \pi_s \left(\sum_{t \in T} \sum_{i \in I} \sum_{j \in J} (c_{ij} - r_j) d_{jts} x_{ijts} \right) & \\
s.t. \quad & \sum_{i \in I} x_{ijts} \leq 1 & j \in J,\ t \in T,\ s \in S \\
& \sum_{j \in J} d_{jts} x_{ijts} \leq q_i y_i & i \in I,\ t \in T,\ s \in S \\
& y_i \in \{0, 1\} & i \in I \\
& x_{ijts} \geq 0 & i \in I,\ j \in J,\ t \in T,\ s \in S
\end{aligned}
$$

Comments

So far we have been concerned only about costs $\rightarrow$ system efficiency.

Is this enough in practice?

What is the function of such a system?

$\downarrow \qquad \downarrow$

Supply the customers!

What is missing?

An effectiveness measure!

Outline

Stochastic demand

We can make use of the service level for measuring effectiveness.

But... how to do it under uncertainty?...

By setting a desirable threshold for the global service level.

↓

Target service level!

α^0 target service level.

h unit cost or financial penalty for staying below the target service level.

Embedding a target service level

Additional decision variables:

α_s = service level achieved under scenario $s \in S$.

Δ_s = service level reduction w.r.t. the target under scenario $s \in S$.

v_{jts} = proportion of unsupplied demand of customer $j \in J$ in period $t \in T$ under scenario $s \in S$.

For every $s \in S$ we have:

$$\alpha_s = 1 - \frac{\sum_{j \in J} \sum_{t \in T} d_{jts} v_{jts}}{\sum_{j \in J} \sum_{t \in T} d_{jts}}$$

$$\Delta_s = \max\{0, \alpha^0 - \alpha_s\}$$

Embedding a target service level

$$
\begin{aligned}
\min \quad & \sum_{i\in I} f_i y_i + \sum_{s\in S} \pi_s \left(h\,\Delta_s + \sum_{t\in T}\sum_{i\in I}\sum_{j\in J} (c_{ij} - r_j) d_{jts} x_{ijts} \right) \\
s.t. \quad & \sum_{i\in I} x_{ijts} + v_{jts} = 1 && j \in J,\ t \in T,\ s \in S \\
& \sum_{j\in J} d_{jts} x_{ijts} \le q_i y_i && i \in I,\ t \in T,\ s \in S \\
& \Delta_s \ge \alpha^0 - \left(1 - \frac{\sum_{j\in J}\sum_{t\in T} d_{jts} v_{jts}}{\sum_{j\in J}\sum_{t\in T} d_{jts}} \right) && s \in S \\
& y_i \in \{0,1\} && i \in I \\
& x_{ijts} \ge 0 && i \in I,\ j \in J,\ t \in T,\ s \in S \\
& v_{jts} \ge 0 && j \in J,\ t \in T,\ s \in S \\
& \Delta_s \ge 0 && s \in S
\end{aligned}
$$

Outline

Capacity disruption

Uncertainty in capacities typically occurs due to some disruption:

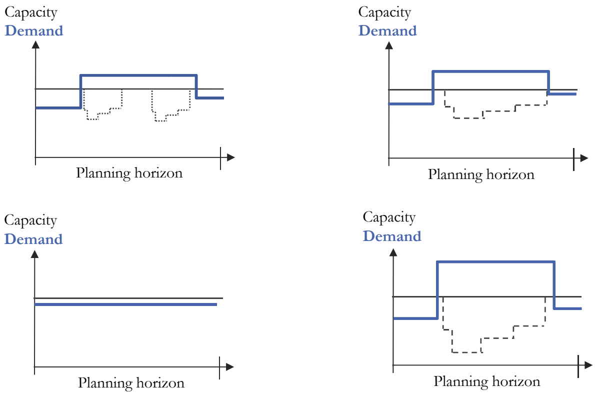
- strike;
- natural disaster;
- man-made disaster (e.g. terrorist attack);
- machine failure;
- short circuit;
- ...

In terms of our modeling setting this corresponds to a reduction in the capacity.

$\surd$ Assume that we can identify a set of scenarios in terms of one or several disruptive triggers and their impact in the operating capacity of the facilities.

$\surd$ If we combine each of these scenarios with each scenario already in S we obtain an extended set of scenarios each defining all the uncertain parameters.

"Extended scenarios"



Anticipating capacity reductions

Uncertainty in capacity can be anticipated by considering options for temporary capacity expansions.

$\downarrow$

Here-and-now decision.

$\downarrow$

A company should decide in advance about possibilities that may need to be activated in case some disruption occurs: preparedness measures.

Additional notation

Parameters driven by possible disruptions:

γ_{its} proportion of the operational capacity of facility $i \in I$ in period $t \in T$ that is available under scenario $s \in S$.

Parameters associated with the preparedness measures:

g_i fixed costs associated with an option contracted for facility $i \in I$ in order to assure a temporary capacity expansion if necessary.

L set of capacity expansion levels available.

Each level determines some (temporary) increase in the operational capacity of a facility.

b_ℓ unit cost associated with capacity expansion $\ell \in L$.

k_ℓ amount of extra capacity associated with capacity expansion $\ell \in L$.

A model for capacity recovery

Additional decision variables:

$$z_i = \begin{cases} 1, & \text{if a capacity expansion option is contracted for facility } i \in I; \\ 0, & \text{otherwise.} \end{cases}$$

$$w_{it\ell s} = \begin{cases} 1, & \text{if expansion level } \ell \in L \text{ is used at facility } i \in I \text{ in period } t \in T \\ & \text{under scenario } s \in S; \\ 0, & \text{otherwise.} \end{cases}$$

A model for capacity recovery

$$\min \quad \sum_{i\in I}(f_i y_i + g_i z_i) + \sum_{s\in S}\pi_s \Bigg[h\,\Delta_s + \sum_{i\in I}\sum_{\ell\in L}\Bigg(b_\ell k_\ell \sum_{t\in T} w_{it\ell s}\Bigg) + \sum_{t\in T}\sum_{i\in I}\sum_{j\in J}(c_{ij} - r_j) d_{jts} x_{ijts} \Bigg]$$

$$s.t. \quad \sum_{i\in I} x_{ijts} + v_{jts} = 1 \qquad j\in J,\ t\in T,\ s\in S$$

$$\sum_{j\in J} d_{jts} x_{ijts} \le \gamma_{its} q_i y_i + \sum_{\ell\in L} k_\ell w_{it\ell s} \qquad i\in I,\ t\in T,\ s\in S$$

$$z_i \le y_i \qquad i\in I$$

A model for capacity recovery

$$
\begin{array}{lll}
s.t. & \displaystyle\sum_{\ell \in L} w_{it\ell s} \leq z_i & i \in I,\ t \in T,\ s \in S \\
& \Delta_s \geq \alpha^0 - \left(1 - \dfrac{\sum_{j \in J} \sum_{t \in T} d_{jts} v_{jts}}{\sum_{j \in J} \sum_{t \in T} d_{jts}}\right) & s \in S \\
& x_{ijts} \geq 0 & i \in I,\ j \in J,\ t \in T,\ s \in S \\
& y_i \in \{0,1\} & i \in I \\
& v_{jts} \geq 0 & j \in,\ t \in T,\ s \in S \\
& \Delta_s \geq 0 & s \in S \\
& z_i \in \{0,1\} & i \in I \\
& w_{it\ell s} \in \{0,1\} & i \in I,\ \ell \in L,\ s \in S
\end{array}
$$

Outline

Model features

The previous stochastic facility location model makes use of all ingredients underlying a recent definition of supply chain risk:

Definition (Heckmann et al., 2018)

Supply chain risk is the time-dependent potential loss for a supply chain in terms of its target values of profitability and functionality evaluated by the decision's maker risk attitude and evoked by uncertain changes of the supply chain and its processes caused by the occurrence of triggering events.

The model can be looked at as a risk-aware capacitated facility location problem.

Model features

Main elements underlying supply chain risk:

- √ time dependency (e.g., disruption profiles);
- √ risk objective (efficiency and effectiveness);
- √ risk attitude (risk neutral?...);
- √ risk exposition (specified by disruptive triggers).

More details:

[Heckmann et al., Omega, 2015]

[Dunke et al., EJOR, 2018]

Model relevance

Is the increased complexity of the presented model compensated by the additional insights provided by the resulting solutions?....

- Value of the stochastic solution?
- The expected value of the perfect information?

Do these values measure the relevance of considering risk?

No!

Model relevance

How to compute a relevant measure?

↓

The single-scenario model does not represent the problem we need to solve if we know the future!

- √ If we know the future, it makes no sense to buy options!
- √ If we know the future, the service level is not uncertain and thus setting a desirable target makes no sense.

What is an adequate deterministic counterpart to our risk-aware model?....

Model relevance

Adequate single-scenario model (for a scenario s):

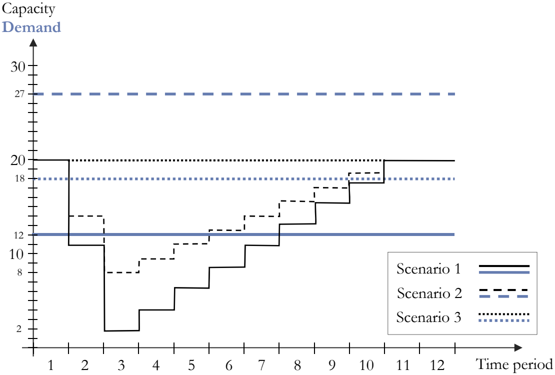
$$
\begin{aligned}
\min \quad & \sum_{i \in I} f_i y_i + \sum_{t \in T} \sum_{j \in J} \hat{h} d_{jts} v_{jts} + \sum_{t \in T} \sum_{i \in I} \sum_{j \in J} (c_{ij} - r_j) d_{jts} x_{ijts} \\
s.t. \quad & \sum_{i \in I} x_{ijts} + v_{jts} = 1 \qquad j \in J,\ t \in T \\
& \sum_{j \in J} d_{jts} x_{ijts} \leq \gamma_{its} q_i y_i \quad i \in I,\ t \in T \\
& y_i \in \{0, 1\} \qquad i \in I \\
& x_{ijts} \geq 0 \qquad i \in I,\ j \in J,\ t \in T
\end{aligned}
$$

Using this model we can compute formulas similar to those used for VSS and EVPI and thus quantify the relevance of capturing stochasticity in our case.

Outline

A small example

2 locations 3 customers



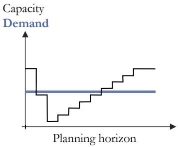
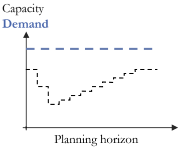
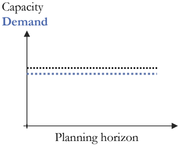
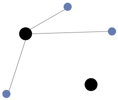
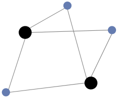
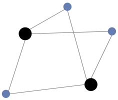
Optimal solution

Both facilities are open: $y_1 = y_2 = 1$.

Expansion options are bought for both facilities: $z_1 = z_2 = 1$.

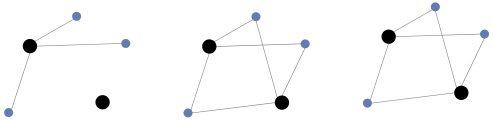
	Time period												Service
	1	2	3	4	5	6	7	8	9	10	11	12	level
Scenario 1													
$k_\ell \times w_{1t\ell 1}$			2	2	2	2							
$k_\ell \times w_{2t\ell 1}$			5	5	2	2	2						
$\sum_{j \in J} v_{jt1}$		1	2.0		0.5								
α_1													0.98
Scenario 2													
$k_\ell \times w_{1t\ell 1}$	5	5	5	10	5	5	5	5	5	2	5	2	
$k_\ell \times w_{2t\ell 1}$	2	5	10	5	10	10	5	5	5	5	2	5	
$\sum_{j \in J} v_{jt2}$		3	4	2.5	1		3	1.5		1.5			
α_2													0.95
Scenario 3													
$k_\ell \times w_{1t\ell 1}$													
$k_\ell \times w_{2t\ell 1}$													
$\sum_{j \in J} v_{jt3}$													
α_3													1.00

The value of a risk-aware solution

	Scenario 1	Scenario 2	Scenario 3
Capacity and demand profiles			
MPSFLP$_s$			
	$\alpha_1 = 0.76$	$\alpha_2 = 0.78$	$\alpha_3 = 1.00$

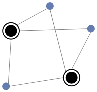
The value of a risk-aware solution

MPSFLP$_s$ Scenario 1 Scenario 2 Scenario 3



$\alpha_1 = 0.76$ $\alpha_2 = 0.78$ $\alpha_3 = 1.00$

CFLP$_{risk}$



$\alpha_1 = 0.95$, $\alpha_2 = 0.949$, $\alpha_3 = 0.95$

The value of a risk-aware solution

Stochastic model

Facilities costs	Scenario 1	Scenario 2	Scenario 3	
Operation	2500	2500	2500	
Expansion	450	1380	300	
Total	2950	3380	2800	
Optimal value				3147

Single scenario problems

Facilities costs	Scenario 1	Scenario 2	Scenario 3
Operation	1000	2500	2500
Expansion	0	0	0
Total	1000	2500	2500
Optimal value	2725	6100	2500

$$0.1 \times 2725 + 0.3 \times 6100 + 0.6 \times 2500 - 3147 = 455,5$$

Outline

A multi-stage stochastic programming model

$$\min \quad \sum_{s\in S} \pi_s \Bigg[\sum_{i\in I} \sum_{t\in T} (f_{it} y_{its} + g_{it} z_{its}) + h^0 \Delta_s^0 + \sum_{t\in T} h_t \Delta_{st} + \sum_{i\in I} \sum_{\ell\in L} \left(b_\ell k_\ell \sum_{t\in T} w_{it\ell} \right) + \sum_{t\in T} \sum_{i\in I} \sum_{j\in J} (c_{ij} - r_j) d_{jts} x_{ijts} \Bigg]$$

$$\begin{aligned}
s.t. \quad & y_{i1s} = y_{i11} && s \in S \setminus \{1\} \\
& z_{i1s} = z_{i11} && s \in S \setminus \{1\} \\
& z_{its} \leq y_{its} && i \in I,\ t \in T,\ s \in S \\
& y_{its} \geq y_{i,t-1,s} && i \in I,\ t \in T \setminus \{1\},\ s \in S \\
& z_{its} \geq z_{i,t-1,s} && i \in I,\ t \in T \setminus \{1\},\ s \in S
\end{aligned}$$

A multi-stage stochastic programming model

$$s.t. \quad \sum_{i \in I} x_{ijts} + v_{jts} = 1 \qquad j \in J,\ t \in T,\ s \in S$$

$$\sum_{j \in J} d_{jts} x_{ijts} \leq \gamma_{its} q_i y_{its} + \sum_{\ell \in L} k_\ell w_{it\ell s} \qquad i \in I,\ t \in T,\ s \in S$$

$$\sum_{\ell \in L} w_{it\ell s} \leq z_{its} \qquad i \in I,\ t \in T,\ s \in S$$

$$\Delta_{ts} \geq \alpha^t - \left(1 - \frac{\sum_{j \in J} d_{jts} v_{jts}}{\sum_{j \in J} d_{jts}}\right) \qquad t \in T,\ s \in S$$

$$\Delta_s^0 \geq \alpha^0 - \left(1 - \frac{\sum_{j \in J} \sum_{t \in T} d_{jts} v_{jts}}{\sum_{j \in J} \sum_{t \in T} d_{jts}}\right) \qquad s \in S$$

Domain constraints: $x_{ijts},\ y_{its},\ v_{jts},\ \Delta_s^0,\ \Delta_{ts},\ z_{its},\ w_{it\ell s}$

Outline

Aspects that have been less treated in the literature:

- The explicit use of service level in the context of multi-period location;
- Rolling horizon planning;
- The value of the multi-period solution;
- The quantification of risk in facility location problems;
- A more comprehensive quantification of the attitude of the decision maker towards risk.
- *Et cetera.*

Some references

M. Albareda-Sambola, A. Alonso-Ayuso, L.F. Escudero, E. Fernández, Y. Hinojosa, C. Pizarro-Romero.
A computational comparison of several formulations for the multi-period incremental service facility location problem.
TOP, 18:62–80, 2010.

J Castro, S. Nasini, F. Saldanha-da-Gama.
A cutting-plane approach for large-scale capacitated multi-period facility location using a specialized interior-point method.
Mathematical Programming A, 163:411–444, 2017.

I. Correia, F. Saldanha-da-Gama.
Facility location under uncertainty.
G. Laporte, S. Nickel, F. Saldanha-da-Gama (editors) *Location Science*, Springer, Berlin Heidelberg, 2015, Chapter 11, pp 289–310.

J. Current, S. Ratick, Ch. ReVelle.
Dynamic facility location when the total number of facilities is uncertain: A decision analysis approach.
European Journal of Operational Research, 110:597–609, 1997.

Some references (cont.)

F. Dunke, S. Nickel.
A general modeling approach to online optimization with lookahead.
Omega, 63:134–153, 2016.

F. Dunke, S. Nickel.
Evaluating the quality of online optimization algorithms by discrete event simulation.
Central European Journal of Operations Research, 25:831–858, 2017.

F. Dunke, S. Nickel.
Day-ahead and online decision-making for collaborative on-site logistics.
Journal of Simulation, DOI: 10.1080/17477778.2018.1485616

F. Dunke, I. Heckmann, S. Nickel, F. Saldanha-da-Gama.
Time traps in supply chains: is optimal still good enough?.
European Journal of Operational Research, 264:813–929, 2018.

I. Heckmann, T. Comes, S. Nickel.
A critical review on supply chain risk — definition, measure and modelingThe curious case of supply chain risk analytics.
Omega, 52:119–132, 2015.

Some references (cont.)

I. Heckmann, S. Nickel, F. Saldanha-da-Gama.
The curious case of supply chain risk analytics.
Submitted for publication.

A. Klose, A. Drexl.
Facility location models for distribution system design.
European Journal of Operational Research, 162:4–29, 2005.

M.T. Melo, S. Nickel, F. Saldanha-da-Gama.
Facility location and supply chain management—A review.
European Journal of Operational Research, 196:401–412, 2009.

S. Nickel, F. Saldanha-da-Gama.
Multi-period facility location.
G. Laporte, S. Nickel, F. Saldanha-da-Gama (editors) *Location Science*, Springer, Berlin Heidelberg, 2015, Chapter 8, pp 177–203.

Logistics Network Design and Facility Location: The value of a multi-period stochastic solution

Francisco Saldanha-da-Gama

Departmento de Estatística e Investigação Operacional,
Centro de Matemática Aplicações Fundamentais e Investigação Operacional,
Faculdade de Ciências, Universidade de Lisboa, Portugal

ORBEL/GOR Joint Annual International Conference

Brussels, Belgium, September 12–14, 2008

Thank you for your attention!